

Proposition for a report in Master Thesis

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The report lies on the study of the following article :

A Very Singular Solution of the Heat Equation with Absorption, H Brezis, L.A. Peletier & D. Terman,
Arch. Rational Mech. Anal. 95 (1986), no. 3, 185–209.

In this article, the authors consider the following equation :

$$(1) \quad \begin{cases} u_t - \Delta u + u^p = 0 & \text{on } \mathbb{R}^N \times (0, \infty) \\ u > 0 & \text{on } \mathbb{R}^N \times (0, \infty) \\ u(x, 0) = c\delta & \text{on } \mathbb{R}^N \end{cases}$$

with $N \geq 1$, $c > 0$ and $\delta(x)$ is the Dirac mass at the origin.

For $p \geq \frac{N+2}{N}$, the only solution must vanish in whole Ω .

For $p < \frac{N+2}{N}$, the behaviour of the solution is like $cE(x, t)$ where E is the fundamental solution of the heat equation.

Using self-similar solutions, the authors found a singular solution of (1) more singular than E . Moreover, they characterize it from the asymptotic point of view.