

## MICROLOCAL ANALYSIS ON THE HEISENBERG GROUP

The aim of this subject is to learn the basis of microlocal analysis on the Heisenberg group. The Heisenberg group is the simplest Lie group and has been a subject of constant interest in Harmonic Analysis. Recent works have been devoted to the development of a microlocal analysis on the Heisenberg group, in relation with what has been done in the Euclidian space since the early seventies. The main idea of microlocal analysis is to take into account information in Fourier variables simultaneously with properties in standard variables. Numerous powerfull tool have been developed in the Euclidian case and have now their counterpart in the Heisenberg group setting.

In this subject, we propose to the student to get familiarized with these aspects of Analysis through the reading of two articles. The first one is devoted to the Heat kernel on the Heisenberg group by Hajer Bahouri and Isabelle Gallagher and will be an introduction to the Heisenberg group. The second one presents an aspect of microlocal analysis in  $\mathbb{R}^n$  : microlocal defect measures. These measures have been introduced simultaneously by Patrick Gérard and Luc Tartar and we propose to read the presentation by Patrick Gérard. For the moment, microlocal defect measures do not exist in the Heisenberg setting. Therefore, the reading of these two articles opens a direction of research.

In a second time and in this direction of research, two articles could be read. Microlocal defect measures on the Euclidian case crucially rely on the existence of a pseudodifferential calculus, therefore the article [4] which defines such a calculus on the Heisenberg group could be usefull. It is also to be noticed that microlocal defect measures allow a good understanding of the ‘div-curl’ lemma in  $\mathbb{R}^n$  and that such a theorem also exists in the Heisenberg group (see [3]), which motivates the introduction of microlocal defect measures on the Heisenberg group.

We propose to the students to ask us for a scanned version of the articles which are not available on the web.

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### **Bibliographie:**

- [1] H. Bahouri et I. Gallagher: The heat kernel and frequency localized functions on the Heisenberg group. (to appear) <http://people.math.jussieu.fr/gallagher/liste-publi.html>
- [2] P. Gérard: Microlocal defect measures. *Comm. Part. Diff. Eq.*, **16**, 1991, p. 1761-1794.

- [3] B. Franchi, N. Tchou, M.C. Tesi: div-curl Type Theorem, H-Convergence, and Stokes Formula in the Heisenberg Group. *Communications in Contemporary Mathematics*, 2006, 8, p. 1-33.
- [4] H. Bahouri, C. Fermanian-Kammerer, I. Gallagher: Phase space analysis and pseudodifferential calculus on the Heisenberg group. <http://arxiv.org/abs/0904.4746>