Aurélien Alvarez
Université d'Orléans, UFR Sciences
Bâtiment de mathématiques - Route de Chartres
B.P. 6759 - 45067 Orléans cedex
France
aurelien.alvarez@univ-orleans.fr

Kazhdan's property (T) and Orbit Equivalence

In the mid 60's, D. Kazhdan defined his property (T) for locally compact groups ([Kaz67]) and used it as a tool to demonstrate that a large class of lattices are finitely generated. Recall that a lattice Γ in a locally compact group G is a discrete subgroup such that the quotient space G/Γ carries a G-invariant probability measure; arithmetic and geometry provide many examples of countable groups which are lattices in semisimple groups, such as the special linear groups $\mathrm{SL}_n(\mathbf{R})$, the symplectic groups $\mathrm{SP}_{2n}(\mathbf{R})$, and various orthogonal or unitary groups. Property (T) was defined in terms of unitary representations, using only a limited representation theoretic background. Later developments showed that it plays an important role in many different subjects ([BHV08]).

Given an action of a countable group on a standard non-atomic probability space by measure preserving transformations, *Orbit Equivalence* aims at understanding the partition of the space into orbits. What kind of information can be recovered from the group and the action just by looking at the equivalence relation on the space whose classes are the orbits of the action ? In the presence of property (T) groups, Furman discovered very strong rigidity phenomena ([Fur99a] and [Fur99b]). The techniques beyond the proofs are in the continuation of the work of Zimmer in the 80's and are beautiful.

Orbit Equivalence has become a very active research area in the last decade and involves techniques from algebraic groups, ergodic theory, von Neumann algebras and combinatorial geometry. I would be pleased to introduce this very nice area to students interested in ergodic theory, groups and geometry, one goal would be to understand the proof of Furman concerning the action of $\mathrm{SL}_3(\mathbf{Z})$ on the torus $\mathbf{T}^3$. Also, it would very interesting to play with numerical experiments to understand the very strong ergodicity of this action. This training course will take place at the University of Orléans, France.

References

[BHV08] B. Bekka, P. de la Harpe, A. Valette – *Kazhdan's property (T)*, Cambridge University Press, (2008).

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[Fur99b] A. Furman – Orbit equivalence rigidity, *Ann. of Math. (2)* **150** (1999), 1083–1108.

[Kaz67] D. Kazhdan – Connection of the dual space of a group with the structure of its closed subgroups, *Func. Anal. Appl.* **1** (1967), 63–65.