Master – Doctorate thesis at the University of Orléans in
General Relativity – Numerical Simulations and Advanced Applied Mathematics

Laboratories: LPC2E (Laboratoire de Physique et Chimie de l'Environnement et de l'Espace), affiliated to OSUC (Observatoire de Sciences de l'Univers en région Centre) / MAPMO (Mathématiques et Applications, Physique Mathématique d'Orléans)

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Post and candidate profiles: The analysis of the relativistic motion and of the gravitational radiation during the capture of stars by supermassive black holes is one of the main targets of the space international mission LISA (Laser Interferometer Space Antenna) planned by ESA and NASA.
The master stage work consists in the utilisation and update of an existing numerical programme or its rewriting for parallel computing. The doctorate work consists in the implementation of the self-consistent prescription for analysis of orbital evolution.
Candidates with knowledge of general relativity, numerical computing and Fortran are strongly preferred, but other motivated candidates should not refrain from applying. Interest in theoretical physics, in astrophysical scenario, with high and sophisticated mathematical content is mandatory. In Orléans, there is the availability of advanced parallel computing systems.

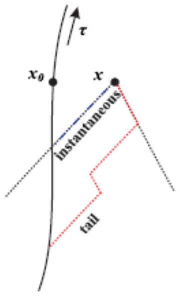
The astrophysical scenario and the detection: EMRI

It is currently believed that most of the galaxies host supermassive black holes in their cores. In this scenario, stars and other compact objects, in the neighbourhood of the central supermassive black hole, spiral-down and plunge in, generating gravitational waves. Furthermore, gravitational waves might be detected when radiated by the Milky Way Sgr*A, a black hole of more than 3 million solar masses in its centre.
The gravitational waveforms of these sources, EMRI (Extreme Mass Ratio Inspiral), are heavily influenced by radiation reaction. Indeed, the relativistic two-body problem implies the emission of radiation that reacts back on the motion (non-radiative back-action shows itself even in Newtonian physics, as the equivalence principle holds as long as the masses of the falling bodies are negligible).

The theoretical physics challenge: the self-force

Black hole perturbations were first dealt in 1957, when a Schwarzschild-Droste black hole was shown to regain stability after undergoing small vibrations about its spherical form, if subjected to a perturbation caused by a small captured mass. In the following forty years, the frequency domain analysis has investigated the captured mass radiating energy (the second time derivative of the quadrupole moment is different from zero), but its motion was still unaffected by the radiation emitted. The breakthroughs arrived thanks to a specifically tailored finite differences method, consisting of the numerical integration of the inhomogeneous wave equation in time domain, and thanks to the determination of the self-force, that is the radiation reaction for point masses.
Before the appearance of the self-force equation and of the regularisation methods, the main theoretical unsolved problem was represented by the infinities of the perturbations at the position of the point mass, represented by a delta distribution.
From 1997, we possess methods for the evaluation in strong field of the self-force for a point mass or particle, to which the captured compact objects are compared, via the conservation of the total stress-energy tensor, or via the matched asymptotic expansion, or via the axiomatic approach, but all yielding the same formal expression, baptised MiSaTaQuWa from the surname first two initials of its discoverers. On the footsteps of Dirac's definition of radiation reaction, in 2003, a fourth approach was presented and later on, other approaches were proposed.
One pictorial description of the self-force refers to a point mass that crosses the curved spacetime and thus generates gravitational waves. These waves are partly radiated to infinity (the instantaneous part) and partly are scattered back by the black hole potential (the non-local part) thus forming tails which impinge on the particle and give origin to the self-force.
Alternatively, the same phenomenon is described by an interaction particle-black hole generating a field which behaves as outgoing radiation in the wave-zone and thereby extracts energy from the particle. In the near-zone, the field acts on the particle and determines the self-force which impedes the particle to move on the geodesic of the background metric.

The self-force is only defined in the harmonic or Lorenz gauge and thus it is not gauge invariant. Being the self-force affected by gauge choice, the equivalence principle allows to find a gauge where the self-force disappears. Again, as in Newtonian physics, such gauge will be dependent of the falling point mass impeding the uniqueness of acceleration.. It should not be overlooked that the self-force is computed in proper time.
The identification of the tail and instantaneous parts was not accompanied by a prescription of cancelation of the divergencies, which indeed arrived three years later thanks to the mode-sum method.



The radiation is going to infinity (instantaneous) or scattered back (tail). The latter determines the self-force. The position of the particle on the worldline τ is z_u, while x is the evaluation point. The self-force is defined for $x \rightarrow z_u$. The self-force determines the deviation from the geodesic equation given by (the star indicates the tail or radiative component of the perturbations $h_{\alpha\beta}$, while $g^{\alpha\beta}$ is the background metric and u^α is the background four-velocity):

$$F^\alpha_{self} = m\frac{Du^\alpha}{d\tau} = \frac{d^2x^\alpha}{d\tau^2} + {}^0\Gamma^\alpha_{\beta\gamma}u^\beta u^\gamma = ma^\alpha_{self} = -m(g^{\alpha\beta}+u^\alpha u^\beta)\left(\nabla_\delta h^*_{\beta\gamma} - \frac{1}{2}\nabla_\beta h^*_{\gamma\delta}\right)u^\gamma u^\delta$$

An alternative pragmatic approach is the direct implementation of the geodesic in the full metric (background + perturbations) in coordinate time and it is coupled to a regularisation by the Riemann-Hurwitz zeta function.

The perspective: the mathematical and numerical method

The two-body relativistic problem poses still formidable challenges even for radial fall and generally whenever adiabaticity can't be evoked. Indeed, adiabatic averaging intervenes if a sufficiently long period, in which energy-momentum balance may be applied, does exist. In such cases the computation of the gravitational radiation equals the orbital energy loss as in the famous binary pulsar 1913+16, which was the object of the Physics Nobel in 1993. But in the strong field curved spacetime, at any time the emitted radiation may backscatter off the spacetime curvature, and interact back with the captured mass later on: the instantaneous conservation of energy is not applicable and the momentary self-force acting on the point mass depends on the point mass entire history.
Thus, the computation and the application of the back-action all along the trajectory and the continuous correction of the background geodesic, it is the only semi-analytic way to determine motion. Non-adiabatic gravitational waveforms are one of the original aims of the self-force community, since they express i) the physics closer to the black hole horizon ii) the most complex trajectories iii) the most tantalising theoretical questions. For the evolution of an orbit, a self-consistent approach has been recommended. Such prescription affirms the greater accuracy of a first order perturbation development along a continuously corrected trajectory as opposed to a higher order perturbation development made on the background geodesic. Self-consistency bypasses the issue of Lorenz gauge relaxation, since at each integration step a new geodesic is found, and consists of a system of three equations to be simultaneously solved, one of which is written above.
The implementation of the self-consistent prescription is under consideration, but far from being gained. At each integration step, for a given number of modes, the process consists of: evaluation of the perturbation functions at the position of the particle by means of retarded Green functions computed in the past light-cone; regularisation by mode-sum or zeta methods; correction of the geodesic and identification of the cell crossed by the point mass; computation of the source term containing delta distributions and its derivatives; reiteration of the above. Computationally and conceptually, it is a formidable challenge, but today one opportunity to study non-adiabatic motion for binary configurations having a small mass ratio.

Bibliography
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