

Anne Estrade  
Professeur  
Université Paris Descartes  
[anne.estrade@univ-paris5.fr](mailto:anne.estrade@univ-paris5.fr)  
tel : 33 (0)1 42 86 21 14  
Head of the project ANR-05-BLAN-0017

## Master internship (M2) proposal

The project ANR-05-BLAN-0017 « mipomodim » (<http://mipomodim.math-info.univ-paris5.fr/>) is concerned with the modelization and the images of porous media. Among this project, a topic is available and tractable for a 2<sup>nd</sup> year master student, to be performed in 3 months during the first semester of 2008.

The internship will take place at the maths laboratory of Université Paris Descartes (MAP5 <http://www.math-info.univ-paris5.fr/map5>), 45 rue des Saints-Pères, in Paris, France. The project could support the travel and stay expenses.

The candidate should be interested in applications and should be able to handle with probability theory and statistics, as well as programming (random Gaussian fields, statistics estimators, numerical integrals, ...).

### Topic: Chord length distribution in a thresholded Gaussian field (numerical study)

The starting point of the study is the paper «Chord-distribution functions and Rice formulae. Application to random media » by Estrade; Iribarren and Kratz (<http://www.math-info.univ-paris5.fr/map5/publis/abstracts07.html#10>).

In this paper the main tools are introduced in order to modelize a porous medium through a thresholded Gaussian field. A Gaussian field is a family of random variables indexed by  $\mathbb{R}^3$  such that every linear combination of a finite number of these variables is Gaussian. In the porous media context, each point of the 3D state space is associated either with the value 1 or the value 0, depending on the presence (matrix) or absence (pore) of material. The corresponding Boolean field is assumed to be given as the indicator function of a Gaussian field  $X$  above a fixed level. When a line is drawn within the medium, one can observe a sequence of alternate intervals either in the matrix part or in the porous part. The chords are defined as the lengths of those intervals.

Using some theorems of renewal theory, and under restricted assumptions, the exact distribution of the chords lengths can be computed. However the formulae are not explicit ones or are numerically hard to be evaluated.

The task of the master student will be first to understand the results and then to simulate thresholded Gaussian fields in order to estimate the chords lengths distribution. The study should conclude with the comparison of theory and simulations/estimations.

**To go further...** The type of technics developed here is part of two maths domains, « mathematical morphology » and « stochastic geometry ». They rely on a strong basis of fundamental mathematics as geometry, topology, differential geometry, measure theory, probability theory. Nevertheless they are hardly connected to applications in geology, petrology, medicine, industrial and medical imaging, networks, ... Furthermore, the increasing faculty of computing leads to the necessity of expanding new tools.