M2 ORLÉANS-HO CHI MINH. A REGRESSION METHOD TO SOLVE BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS

FRANÇOIS DELARUE, UNIVERSITÉ PARIS DIDEROT (PARIS 7).

A Backward Stochastic Differential Equation (BSDE in short) is a stochastic differential equation with a terminal boundary condition.

Given a Brownian motion $(B_t)_{t\geq 0}$ on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$, it has the form

$$\forall t \in [0,T],\ Y_t = \xi + \int_t^T f(s, Y_s, Z_s)ds - \int_t^T Z_s dB_s.$$

Above, T stands for the horizon, f for the driver and ξ for the boundary condition. The boundary condition ξ is random: it is $\mathcal{F}_T$ measurable, where $(\mathcal{F}_t)_{t\geq 0}$ stands for the filtration generated by the Brownian motion, augmented with null sets. Similarly, the driver f may be random (progressively with respect to the filtration).

If exists, the pair (Y, Z) is a solution of the BSDE. At any time $t \in [0,T]$, Y_t has to be $\mathcal{F}_t$-measurable. This constraint explains the role of the process $(Z_t)_{0\leq t\leq T}$: it brings back the boundary condition in the right σ-algebra.

Solvability properties of BSDEs have been widely investigated for fifteen years. (See [3] for the original paper.) When the square norm of ξ is integrable and f is Lipschitz continuous in (y, z), the above equation is uniquely solvable.

From a practical point of view, BSDEs appear in numerous problems in finance. The standard theory of contingent claim in a complete market may be expressed in terms of BSDEs. The price of the claim ξ at maturity T is given, for a suitable linear f, by the value Y_0 of the process Y at time 0. When trading constraints on some assets are imposed, the related BSDE is non-linear. In incomplete markets, hedging strategies may be also expressed in terms of BSDEs. (See [1] and [4]).

For these reasons, numerical methods for BSDEs have been developed for ten years. The aim of the internship is to study one of these methods, based on a regression procedure. (See [2]).

References

[1] El Karoui, N.; Peng, S.; Quenez, M. C. Backward stochastic differential equations in finance. Math. Finance 7 (1997), 1–71.

[2] Gobet, E.; Lemor, J.-P.; Warin, Xavier. A regression-based Monte Carlo method to solve backward stochastic differential equations. Ann. Appl. Probab. 15 (2005), 2172–2202.

[3] Pardoux, É.; Peng, S. G. Adapted solution of a backward stochastic differential equation. Systems Control Lett. 14 (1990), 55–61.

[4] Rouge, R.; El Karoui, N. Pricing via utility maximization and entropy. INFORMS Applied Probability Conference (Ulm, 1999). Math. Finance 10 (2000), 259–276.