Sujet de stage pour le Master de Mathématiques
Orléans – Ho Chi Minh City

Titre : *Equation des ondes non linéaire* [**Title :** *Nonlinear wave equation*]
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Abstract :
Nonlinear evolution equations have been intensively studied since the eighties. We shall consider one of the main examples, namely the nonlinear wave equation

$$\text{(NLW)} \qquad \begin{cases} \partial_t^2 u(t,x) - \Delta_x u(t,x) = \text{const.}\,|u(t,x)|^\alpha \\ u(0,x) = f(x)\,,\ \partial_t u(0,x) = g(x) \end{cases}$$

in $\mathbb{R}^n$. After several attempts and partial results, Georgiev, Lindblat & Sogge [GLS] proved global wellposedness of (NLW) in all dimensions $n \geq 2$ (existence and uniqueness of solutions in suitable function spaces) for small initial data and α larger than a critical exponent $\alpha(n)$. Tataru [Tat] obtained shortly afterwards a striking simplification for the range $\alpha(n) < \alpha \leq \frac{n+3}{n-1}$ by transferring the problem from the euclidean space $\mathbb{R}^n$ to the hyperbolic space $\mathbb{H}^n$ and by taking advantage of the better behavior of the wave equation in negative curvature. The resulting proof is a beautiful blend of techniques borrowed from PDE, harmonic analysis, and differential geometry.

The present project consists in understanding the paper [Tat], which is rather sketchy, and all the tools involved. Investigating dispersive equations on manifolds is a long term project, which has recently started in Orléans and will be the subject of a forthcoming workshop in Orléans during the seond week of April 2008.

References :

[GLS] V. Georgiev, H. Lindblad, C. Sogge : *Weighted Strichartz estimates and global existence for semilinear wave equations*, Amer. J. Math. 119 (1997), 1291–1319

[Tat] D. Tataru : *Strichartz estimates in hyperbolic space and global existence for the semilinear wave equation*, Trans. Amer. Math. Soc. 353 (2001), 785–807

Other references:

[C] T. Cazenave: *Semilinear Schrödinger equations*, Courant Lect. Notes Math. 10, Amer. Math. Soc. (2003), 323 pp.

[G] V. Georgiev: *Semilinear hyperbolic equations*, Math. Soc. Japan Memoirs 7 (2005), 209 pp.

[Tao] T. Tao: *Nonlinear dispersive equations* (*local and global analysis*), CBMS Reg. Conf. Ser. Math. 106, Amer. Math. Soc. (2006), 373 pp.

A huge amount of material is also available

- on the web site *Dispersive PDE Wiki*
  http://tosio.math.toronto.edu/wiki/index.php/Main_Page
- on the web pages of Terence Tao
  http://www.math.ucla.edu/∼tao/
  → preprints/pde.html