Rectifiability and Löwner Processes

The Löwner differential equation(s) describe planar growing processes. They allow to establish a one-to-one correspondance between increasing one-parameter families of compact sets and one parameter families of positive measures on the circle. The historical case is the case where $\mu_t = \delta_{\xi(t)}$. The function $\xi : [0, +\infty[\to \{|z| = 1\}$ is called the driving function. Originally introduced to solve a problem in geometric function theory, this equation has been more or less forgotten until its spectaular rebirth in the SLE theory originated by Oded Schramm. His idea was to take as a driving function $\xi(t) = \sqrt{\kappa} B_t$ where κ is a positive real and B_t is a $1D$-Brownian motion. Together with S.Rohde, they have discovered the existence of a phase transition in κ. The growing cluster is a simple curve if and only if $\kappa \leq 4$. This result revived the deterministic Löwner equation for which the relation between the smoothness of ξ and of the cluster has bizarrely not been studied very much. Marshall and Rohde have shown that small norm in Hölder class $1/2$ is essentially the sharp condition for the cluster to be a growing simple curve. Joan Lind precised this by proving that it suffices that the norm is less than 4.
In this internship the candidate will study necessary and/or sufficient conditions for the driving function in order for the growing cluster to be a rectifiable simple curve. He or she will investigate in particular Sobolev spaces that are natural in this context in the sense that they contain $1/2$-Hölder functions.

References:

Peter Duren: Univalent functions (Springer).
Lars Ahlfors: Conformal Invariants (Mc Graw Hill).
S.Rohde, O.Schramm: Basic properties of SLE processes.arXiv:math.PR/0106036 D.Marshall, S.Rohde: The Löwner differential equation and slit mappings.