Trace for solutions of a degenerate parabolic equation

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The classical problem was initiated by Fatou : assume that u is a harmonic function on the unit ball of $I\!R^n$, what can we say about the boundary value of u ?

This problem was generalized for solutions of many different partial equations and by a lot of authors like H. Brezis, Sh. Kamin, M. Marcus, L. Véron...

The candidate will have to understand the paper of Sh. Kamin, M.-A. Pozio and A. Tesei : Admissible conditions for parabolic equations degenerating at infinity. They consider the parabolic Cauchy problem :

$$\begin{cases} \rho \partial_t u = \Delta u \quad \text{in} \quad I\!R^n \times I\!R_+ \\ u = u_0 \quad \text{in} \quad I\!R^n \times \{0\} \end{cases} \tag{1}$$

where Δ denotes the Laplace operator in $I\!R^n$ $\left(\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2} \right)$, under the assumptions :

$$\begin{cases} \rho \in C^{1+\alpha}_{\text{loc}}(I\!R^n), \quad \rho > 0 \quad \text{in} \quad I\!R^n \\ u_0 \in C^{\alpha}_{\text{loc}}(I\!R^n) \cap L^\infty(I\!R^n), \quad \alpha \in (0,1). \end{cases}$$

In particular, they prove that if $\Gamma * \rho \in L^\infty(I\!R^n)$ where Γ denotes the fundamental solution of the Laplace equation in $I\!R^n$, then for any bounded solution u of (1), there exists a Lipschitz function A in $\overline{I\!R}_+$ with $A(0) = 0$ such that

$$\lim_{R \to +\infty} \frac{1}{|\partial B_R|} \int_{\partial B_R} \left(\int_0^t u(x,s) ds \right) d\sigma = A(t) \ .$$

This is in this sens that we say that u has a trace at infinity.

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