Ice formation in the Artic during summer: False-bottoms
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1. This model describes the simultaneous growth and ablation of a layer of ice between an under-ice melt pond and underlying ocean.

First, meltwater collects in surface melt ponds and this melt water can be retained under thin ice or in bottom depressions, leading to fresh water lenses calls " under-ice melt ponds". At the interface between this (light) fresh water and the underlying (dense) salt water, double-diffusive convection of heat and soft occurs, leading to the formation of underwater ice called "False-bottoms".

Such "False-bottoms" are the only significant source of ice formation in the Artic during summer.

2. So we have here 3 environments: The ocean (oc), the ice false-bottoms (Fb) and the fresh water (wa).

i) Interface(Fb − Oc)

At the interface ice and the ocean we have if $h_0(t)$ is the free boundary

(1) $\rho L h_0'(t) = \lambda_I \frac{\partial T}{\partial z}|_{Fb} - \lambda_O \frac{\partial T}{\partial z}|_{Oc}$

(1) is the Stefan condition for the heat balance at the interface, $h_0'(t)$ representing the ablation of the ice.

In (1) $T(z,t)$ is the temperature, z the vertical coordinate, L the latent heat fusion and ρ the density of the ice.

λ_I and λ_O are the thermal conductivity, the subscripts I and O refer.

At the same interface we have for the salinity the conservation relationship:

(2) $S_0 h_0'(t) = -D \frac{\partial S}{\partial z}|_{Oc}$

where $S_0 = S(h_0, t)$ is the salinity of the ocean at the interface and D the molecular diffusivity of salt in water.

The interface temperature $T_0 = T_0(, h_0, t)$ and salinity S_0 are connected via freezing-point relationship

(3) $T_0 = -mS_0, \ m > 0$

ii)Diffusion equations in ice and ocean

Equations (1)-(3) form a closed system in connection with the diffusion equations for heat and salt in the ice and in the ocean

(4) $\frac{\partial T}{\partial t} = D_I \frac{\partial^2 T}{\partial z^2}$ (ice)

and in the ocean

(5) $\frac{\partial T}{\partial t} = D_0 \frac{\partial^2 T}{\partial z^2}$

(6) $\frac{\partial S}{\partial t} = D \frac{\partial^2 S}{\partial z^2}$

where D_I, D_O are the thermal diffusivity.

iii) Interface(Fb − wa):

The Stefan condition at he upper surface of the false-bottoms that i.e. at the interface between the ice and fresh water is

(7) $\rho_I L h_u'(t) = \lambda_I \frac{\partial T}{\partial z}|_{Fb}$

where $h_u(t)$ is the position of the upper intersurface.

3. The problem in one dim is to determine numerically the free heights $h_0(t), \ h_u(t)$, the initial salinity $S_0 = S(h_0, t)$ and the initial temperature $T_0 = T(h_0, t)$ for given $S_{-\infty}$ and $T_{-\infty}$.

Describe in 2D this problem and solve it numerically.