Modified method for an inhomogeneous backward heat problem in 2D

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There are several important ill-posed problems for parabolic equations. A classical example is the backward heat equation. In other words it may be possible to specify the temperature distribution at a particular time, say $t = 1$ and from this data the question arises as to whether the temperature distribution at any earlier time $0 < t < 1$ can be retrieved.

Concretely we will consider the following two-dimensional problem

$$u_t - \Delta u = f(x, y, t),\ (x, y) \in \mathbb{R} \times \mathbb{R},\ t \in (0, 1) \tag{1}$$

$$u(x, y, 1) = \varphi(x, y),\ (x, y) \in \mathbb{R} \times \mathbb{R} \tag{2}$$

We want to retrieve the temperature distribution $u(x, y, t)$ for $0 \leq t < 1$. Of course since the data φ is based on physical observations, there will be measurements errors, and we would actually have as data some function φ_ϵ for which $||\varphi(.) - \varphi_\epsilon(.)|| \leq \epsilon$, where $||.||$ denotes the L^2-norm, the constant $\epsilon > 0$ represents a bound on the measurement error.

In order to obtain stable approximation we shall consider the following problem

$$v_t - \Delta v = \mu(\epsilon)\Delta v_t + f(x, y, t),\ (x, y) \in \mathbb{R} \times \mathbb{R},\ t \in (0, 1) \tag{3}$$

$$v(x, y, 1) = \varphi_\epsilon(x, y),\ (x, y) \in \mathbb{R} \times \mathbb{R} \tag{4}$$

which we learned from Elden [1].

Formal solutions of initial value problems associated to the problem (3), (4) have recently been presented in Lesnic [2].

So the main objective of this investigation is to find out how well (3), (4) approximates (1), (2) and construct by means of Fourier transform technique some valuable stability estimates.

Some examples will be given to illustrate the usefulness of this method.

Comparisons with another approach of this ill posed-problem i.e. by means of domain truncature and regularization could also be studied. See for instance [3]

[1]- L. Elden, Approximations for a Cauchy problem for the heat equation, *Inverse Problems, 3 (1987), 263-273.*

[2]- D.Lesnic, Heat conduction with mixed derivatives, *Int.Computer Math., 81 (2004), 971-977.*

[3]-Nguyen Cam, Nguyen Van Nhan & A.Pham Ngoc Dinh, The Backward heat equation: Regularization by Cardinal Series, *Archives of Inequalities and Applications, 2 (2004), 355-363.*