Sujet de stage pour le Master de Mathématiques
Orléans – Ho Chi Minh City

Title : *Modelization of water filtration in porous media*

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Abstract : Understanding the quantitative behavior of the movement of liquids in satured and insatured porous media is one of the main goals of fluid mechanics.

In 1931, an accurate model which describes the flow of water in a porous medium by means of a nonlinear heat equation has been proposed by Richards. Denoting with $\theta = \theta(t,x)$ the ratio between the volume of water in an elementary volume of the porous medium and the volume of the elementary volume of the porous medium, Richards' equation reads, in the space-time variables $(t,x) \in \mathbf{R}^+ \times \mathbf{R}^3$:

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K(\theta) \nabla \Psi(\theta)) + \frac{\partial}{\partial x_3}(K(\theta)) + r, \tag{1}$$

where K is the hydraulic conductivity, Ψ is the pressure potential, x_3 is the elevation above a vertical datum of planar coordinates (x_1, x_2) and r is a constant.

Richards' equation has nevertheless two main drawbacks: first, the fact that the flux on the surface is discontinuous could seem not realistic; second, accurate numerical schemes for Richard's equation are quite time-consuming.

A much simpler model, proposed in 1911, is given by the Green-Ampt equation, which is an equation for the total depth of infiltration F:

$$F = \frac{K_s S_w (\theta_s - \theta_i)}{i - K_s} \tag{2}$$

where θ_s and θ_i are the saturated and initial volumetric water contents, respectively, S_w is the soil water suction (negative pressure) at the wetting front, i is rainfall intensity and K_s is the saturated hydraulic conductivity. This model is based on the assumptions that the wetting front advances at constant rate (and is well-defined); the volumetric water contents remain constant above and below

the wetting front as it moves and, finally, that the soil water suction below the wetting front remains constant.

The present project consists in deducing under what kind of hypothesis it is possible to derive, at least at a formal level, the Green-Ampt equation (2) starting from Richards' equation (1). The first part of this training course will consist in the study of particular solutions, called travelling wave, of the Richards equation. The study will also require some knowledge in scientific computing (e.g. using scilab software) for numerical comparisons of the models.

This proposal is part of a long term project called METHODE which has recently started in Orléans involving also italian partners on the modelling of the behavior of liquids in porous materials.
See http://hydram.epfl.ch/e-drologie/chapitres/chapitre5/chapitre5.html for an introduction of these models (in french).