Sujet de stage pour le Master de Mathématiques
Orléans

Titre : *Solutions analytiques pour les équations de Saint-Venant*
[**Title :** *Analytical solutions for the shallow water equations*]

Responsables : Stéphane Cordier, Olivier Delestre & François James
Coordonnées : Université d'Orléans & CNRS
Fédération Denis Poisson (FR 2964), Laboratoire MAPMO (UMR 6628)
Bâtiment de Mathématiques, BP 6759, 45067 Orléans cedex 2, France
Web : http://www.fdpoisson.org/ & http://www.univ–orleans.fr/mapmo/
Contact email : cordier@math.cnrs.fr

Abstract :
The shallow water (or Saint-Venant) equations are hyperbolic partial differential equations which are used to simulate water flows when the depth of water is not too large. In one space dimension, they read

$$\begin{cases} \partial_t h + \partial_x(hu) = 0 \\ \partial_t(hu) + \partial_x(hu^2 + gh^2/2) + h\partial_x Z = -ghS_f \end{cases},$$

where $h(t,x)$ is the water height, $u(t,x)$ the flow velocity, $z(x)$ the topography, S_f a friction term.

Searching for analytical solutions is important, in order to have a better insight of the problem, and also to obtain test cases for general numerical methods (e.g. finite volumes [B]). Several configurations are known, but they do not take into account friction terms. The aim of this project is, starting from these classical tests ([GHS], [H], [VCEL]), to proceed towards new analytical solutions to this system, including friction terms.

Some abilities in using formal calculus software (such as Maple) are expected since the complete formulation of these analytical solutions contains nonlinear algebraic equations, which have to be solved, for instance by formal calculus.

This training course is part of a research program, called METHODE, funded by the National Research Agency in France (ANR).
See http://www.univ–orleans.fr/mapmo/methode for more details on this project.

References :

[GHS] T. Gallouët, J.-M. Hérard, N. Seguin, Some Godunov schemes to compute shallow-water equations with topography, *Computers & Fluids* **32**, (2003).
[H] J.M. Hervouet, *Hydrodynamique des écoulements à surface libre, modélisation numérique avec la méthode des éléments finis*, Presses des Ponts et Chaussées, (2003).
[VCEL] P.-L. Viollet, J.-P. Chabard, P. Esposito, D. Laurence, *Mécanique des fluides appliquée*,Presses des Ponts et Chaussées, (1998).
[B] F. Bouchut, *Nonlinear stability of finite volume methods for hyperbolic conservation laws, and well-balanced schemes for sources*, Birkhauser (2004).