Sujet de stage pour le Master de Mathématiques
Orléans – Ho Chi Minh City

Titre : *Existence and positivity of equilibrium states for a PDE arising in neuroscience*

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Subject:
We are interested in the study of the evolution of two populations of neurons. Their behaviour is described in terms of the evolution of respective frequencies, ν_1 and ν_2 whose dynamics is modelled by coupled stochastic ordinary differential equations (see [Deco]). This study shows the existence of an equilibrium state and suggests the existence of a stationary solution for the associated PDE (Partial Differential Equation, of Fokker-Planck or parabolic type), describing the evolution in time of the distribution function $f(t, \nu_1, \nu_2)$:

$$\partial_t f + \nabla \cdot (Ff + \beta^2 \nabla f) = 0,$$

where $F = F(\nu_1, \nu_2)$ is a vector function given by:

$$F_i = -\nu_i + \varphi(\lambda_i + a_i \nu_1 + b_i \nu_2), \quad i = 1, 2$$

with a_i, b_i, λ_i, for $i = 1, 2$, are constants given by the modelling and represents the interaction between the two neurone population. Moreover, φ is a sigmoidal function defined on $x \in \mathbb{R}$ by $\varphi(x) = \frac{\nu_c}{1+\exp(-\alpha(\frac{x}{\nu_c}-1))}$, with $\nu_c \in \mathbb{R}$ and α are physical constant.

The first part of this training course is to prove the existence and positivity of solution at the equilibrium for the FP equation. When F derives form a potential (i.e. $F = \nabla V$), we compute an explicit formulation for the equilibrium state f_{eq}. In our case F is not a potential and we shall apply classical theorems (Fredholm alternative and Krein-Rutman theorem, see [Brezis]) from elliptic operators analysis, in order to achieve our goal. Second, using entropy methods, we will prove that the solution of the above PDE converges in large time towards the obtained equilibrium state.

This will requires skills in mathematical analysis and more precisely in functionnal and PDE analysis.

References:
[Brezis] H.Brezis, *Anlyses fonctionnelle-Théorie et applications*, Collection Mathématiques appliquées pour la maîtirse, Ed.Masson (1983)
[Deco] G.Deco, D.Marti, *Deterministic analysis of stochastic bifurcations in multistable neurodynamical system*, Biol. Cybern (2007), V.96, pp. 487-496