Sujet de stage pour le Master de Mathématiques
Orléans – Ho Chi Minh City

Titre : *Modelling of pollutants*
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Subject:

The main objective of this project is to work on mathematical models and to develop numerical methods for the modelling of pollutants in water flows. More precisely, we are interested in the study of the evolution of particles of pollutants interacting with a fluid using both collisional kinetic theory and stochastic differential equations.

We shall assume that the velocity of the fluid is known (as a vector field u depending of space,x, and time, t variable). The position X and velocity V of these particles are governed by the stochastic differential equation of the form

$$dX = V dt, \quad dV = \nu(u(X,t) - V) + \beta dB_t,$$

where dB_t is a Brownian motion.

The distribution function $f(x, v, t)$ (which represents the probability to find a particle at time t, position x and velocity v) of these particles is given by a PDE (Partial Differential Equation), of Fokker-Planck or parabolic type,

$$\partial_t f + v.\nabla_x f + \nabla_v \cdot (\nu(u - v)f + \beta^2 \nabla_v f) = 0.$$

The first part of this training course is to study the PDE and to perform numerical simulations (using Monte Carlo method i.e. going back to the underlying stochastic differential equation) for suitable initial and boundary conditions.

In a second part, we will study the modelling of sedimentation i.e. assuming the particles stop when they encounter some obstacles like the frontier of the domain.

This will require skills in mathematical analysis (PDE, probability) and also in numerical simulations.

References:
Cédric Villani A Review of Mathematical Topics in Collisional Kinetic Theory, in Handbook of Mathematical Fluid Dynamics, S. Friedlander and D. Serre, Eds, Elsevier, 2002