Short description of the project
"Diffusion in irregularly-shaped domains generated by conformal mapping with laculary series"
(supervised by M. Zinsmeister and D. Grebenkov, 06/02/2009)

Fractals are often considered as a paradigm of a complex shape. On one hand, fractals exhibit geometrical complexity at any length scale. On the other hand, a hierarcical structure (such as self-similarity or self-affinity) allows for advanced mathematical analysis and efficient numerical simulations. For planar shapes, conformal mapping is a powerful tool. For most classical fractal shapes (e.g., von Koch snowflakes), finding an appropriate conformal mapping is a difficult numerical problem. For this reason, it is attractive to investigate irregularly-shaped domains which are generated by explicit conformal maps with lacunary series. Let $f(z)$ be a complex-valued function defined as a lacunary series

$$f(z) = \sum_{k=0}^{\infty} a_k z^{2^k} \tag{1}$$

with coefficients a_k (e.g., $a_k = 1$) that ensure absolute convergence inside the unit disk $|z| < 1$. We define a conformal map

$$\varphi(z) = \int_0^z d\zeta \; e^{bf(\zeta)} \tag{2}$$

where b is a small positive parameter. For $b = 0$, we have the conformal mapping from the unit disk onto the unit disk. For a strictly positive b, the unit disk is conformally mapped onto an irregularly-shaped domain whose fractal dimension is larger than 1. Since the numerical computation of $f(z)$ is extremely rapid, these domains are particularly attractive for studying diffusion.

The aim of the project is to generate such quasi-von Koch domains and to investigate their geometrical and transport properties. This project consists in two parts: theoretical study of these curves by means of complex analysis (supervised by M. Zinsmeister) and numerical study (supervised by D. Grebenkov). For the numerical study, an algorithm for generating these domains should first be implemented. After that, a number of questions arise:

1. How the shape of the domain (e.g., its fractal dimension) depends on b, as well as on the choice of coefficients a_k?

2. What are the properties of the harmonic measure on these domains?

3. What is the Laplace operator eigenbasis?

References

[1] M. Brady, C. Pozrikidis, "Diffusive transport across irregular and fractal walls", Proc. R. Soc. Lond A **442**, 571 (1993).