Proposition de stage de Master 2
ANALYSE MATHÉMATIQUE ET APPLICATIONS
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CLASSICAL AND RECENT APPROACHES TO THE MEAN ERGODIC THEOREM

The Mean Ergodic Theorem is a classical result due to John von Neumann. This theorem describes the convergence of Cesàro averages of the powers of a linear operator on a Hilbert space. Von Neumann's proof was based on spectral theory of operators, but very short geometrical proofs are now known.

Ergodic Theory studies the behaviour of iterates of measure preserving transformations of a measure space. This is a natural setting for statement and applications of the Mean Ergodic Theorem. Any basic book on Ergodic Theory, and a lot of Probability Theory exposition books propose good introduction to this subject. For a very brief introduction and some references see the electronic encyclopedia Wikipedia page on "Ergodic Theory".

It is an interesting application of Banach-Steinhaus Theorem (or "uniform boundedness principle") that *there is no speed of convergence in the mean ergodic theorem for measure preserving transformations.* See for example [1].

Recently, in [2], Terence Tao proposed a new "finitary" approach to the ergodic theorem. Tao's article is too complicated to be completely understood in a few weeks, but there is useful and not too difficult information to extract from it. The finitary version of the Mean Ergodic Theorem gives a quantitative information on the convergence, which can be transfered to any measure preserving dynamical system. Thus in a certain sense, there is a universal "rate" of convergence.

The project of master's report includes the exposition of the classical Mean Ergodic Theorem and the description of Tao's approach in the context of a single measure preserving transformation.

[1] U. Krengel, *Ergodic Theorem* (notes on Section 1.2 pages 14-15 in 1985 edition). de Gruyter Studies in Mathematics, 6. Walter de Gruyter and Co., Berlin, 1985.

[2] T. Tao, Norm convergence of multiple ergodic averages for commuting transformations. Ergodic Theory Dynam. Systems 28 (2008), no. 2, 657-688.

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