POSSIBLE TOPICS FOR A MASTER'S THESIS WORK

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1. Word hyperbolic groups

A group G is *of finite type* if it admits a finite generating set. We associate to a groups G and a finite generating set S a graph, the *Cayley graph*, whose vertices are the elements of the groups, and two vertices have an edge in-between if they differ by an element in the generating set S. This graph allows us to endow G with a metric (the graph metric). When each triple defines a uniformly thin triangle, one says that the group is *word hyperbolic*. This definition is independent of the generating set.

In this work, the student will be:

(1) Explaining the history and the motivations of this notion.
(2) Giving some explicit examples.
(3) Explain the proof of the so-called "Morse lemma", that quasi-geodesic travel close to geodesics.
(4) State a few corollaries that the "Morse lemma" implies.

Reference:

- H. Short (edit.) : *MSRI Notes on Hyperbolic groups*, Group Theory from a Geometrical Viewpoint (E. Ghys, A. Haefliger, A. Verjovsky, ed) Proc. ICTP Trieste 1990, World Scientific, Singapore, 1991, 3–64.

`http://www.latp.univ-mrs.fr/~hamish/Papers/MSRInotes2004.pdf`

2. Amenable groups

For a topological group G endowed with a Haar measure μ, we denote by $L^\infty(G,\mu)$ the Banach space of all essentially bounded functions $G \to \mathbf{R}$. A group G is called *amenable* if $L^\infty(G,\mu)$ carry a left-invariant mean, that is a left-invariant linear functional mapping the constant function to its value, and non-negative functions to a positive number. There are many equivalent definitions of amenability, and in this work the student will be:

(1) Explaining the history and the motivations of this notion (the Banach-Tarski paradox).

(2) Giving some explicit examples and non-examples.
(3) Choose a few characterizations of amenability, and explain the equivalences.

References:

- F.P. Greenleaf, *Invariant Means on Topological Groups and Their Applications*, Van Nostrand Reinhold (1969).
- V. Runde *Lectures on Amenability*, Lecture Notes in Mathematics 1774, Springer (2002).

3. Property (T) and aTmenability

A group G has *property (T)* if any affine action by isometries on a Hilbert space has a globally fixed point. On the other hand, a group is called *aTmenable* if it admits a proper action on a Hilbert space. Those notions have several characterizations, and in this work the student will be:

(1) Explaining the history and the motivations of these two notions.
(2) Giving some explicit examples and non-examples.
(3) Working out explicitly why $SL_3(\mathbf{R})$ has property (T).

References:

- B. Bekka, P. de la Harpe and A. Valette *Kazhdan's property (T)*, New Mathematical Monographs, 11, Cambridge University Press, ISBN 978-0-521-88720-5
- P.-A. Cherix, M. Cowling, P. Jolissaint, P. Julg, and A. Valette. *Groups with the Haagerup Property (Gromov's a-T-menability)* Published by Birkhäuser, 2001 ISBN 3764365986

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