

Laplace operator eigenfunctions in complex geometries

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Diffusive transport in complex geometries is ubiquitous in nature and industry: biochemical regulation in cells, diffusion-limited reactions in catalysis, to name a few. The interplay between geometrical complexity and stochastic character of diffusion remains poorly understood, especially in three dimensions. A spectral formulation of diffusion problems is a fundamental way to reveal the role of the geometry and to explain various striking phenomena observed in experiments. In this light, restricted diffusion in complex geometries can be investigated through the properties of the Laplace operator eigenfunctions.

The first part of this project consists in a numerical computation of the eigenfunctions in several model geometries: von Koch snowflakes, random packs of disks and spheres, and 3d labyrinth structures. In the second part, the properties of these eigenfunctions have to be thoroughly investigated, in particular, an emergence of spatially localized eigenmodes and its relation to geometrical complexity. In the third part, the role of the localized eigenmodes for various diffusive phenomena has to be revealed, with special focus on nuclear magnetic resonance applications (matrix formalism).

The project necessarily relies on using theoretical and numerical tools. A basic knowledge in mathematical physics, spectral analysis, probability theory and/or partial differential equations is recommended. Programming and computer skills are particularly welcome. The internship can eventually result in a PhD.

References:

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- D. S. Grebenkov, Laplacian Eigenfunctions in NMR I. A Numerical Tool, *Conc. Magn. Reson. A* 32, 277-301 (2008).
- D. S. Grebenkov, Residence times and other functionals of reflected Brownian motion, *Phys. Rev. E* 76, 041139 (2007).