

1. Let D be the open bounded smooth subset in \mathbb{R}^n , α and β be different non-null real numbers, and u and v be in $W_0^{1,2}(D) \setminus \{0\}$ such that $\Delta u = \alpha u$ and $\Delta v = \beta v$ in weak solutions sense. Prove that

$$\int_{\Omega} \nabla u \cdot \nabla v dx = \int_{\Omega} uv dx = 0.$$

2. Let D be the open bounded smooth subset in \mathbb{R}^n . Prove that there is a weak solution in $W_0^{1,2}(D)$ to following equation

$$\Delta u + \cos u = 0.$$

3. Let D be the open bounded smooth subset in \mathbb{R}^3 . Prove that there is a weak solution in $W_0^{1,2}(D)$ to following equation

$$\Delta u - u = 1.$$