

## Học phần mới

### Từ Tối ưu Lồi đến Tối ưu không Lồi

Giáo sư Phạm Đình Tảo, INSA-Rouen, Pháp

Số tiết: 30 tiết

Thời gian học: 12/1/2009 – 18/1/2009 (một buổi / ngày)

Thời gian học để trang bị các kiến thức cơ bản: 3/1/2009 – 9/1/2009 (không học ngày chủ nhật) do giáo viên Nguyễn Thị Thu Vân phụ trách.

**Tóm tắt môn học:** Môn học này gồm có 2 phần A và B. Phần A: trình bày một số kiến thức cơ bản của Tối ưu lồi để làm nền tảng cho phần B. Phần B: mô hình hoá, tối ưu không lồi và tối ưu toàn cục. Các phương pháp trong Tối ưu không lồi trong phần B được áp dụng để giải các bài toán thực tế trình bày trong môn học của Giáo sư Lê Thị Hoài An.

Môn học này được giảng dạy bởi giáo sư Phạm Đình Tảo, một trong những chuyên gia đầu ngành trong lĩnh vực Tối ưu.

Đối tượng tham gia: tất cả các sinh viên đại học, học viên cao học.

**Các môn học bắt buộc trước:** không

**Nội dung môn học:**

#### **A. CONVEX OPTIMIZATION : Basic tools of modern convex analysis**

##### **I. Convex sets and convex functions taking the infinity value**

Operations preserving convexity

Generating functional operations of convex functions Epigraph of a convex function. Infimal convolution. Convex hull of a set and convex hull of a function.

##### **II. Topological properties for sets and functions**

Relative interior of a convex set. Lower semicontinuity (lsc) functional. Intérieur relatif d'un ensemble convexe. Semi-continuité inférieure (sci) d'une fonction convexe. Lsc-regularized and  $\cong$ -regularized of a convex function.

Asymptote cone of a convex set and characterization of a bounded convex set.

Function asymptote of a convex function. Continuity of a convex function.

##### **III. Duality for sets and functions**

Separation of convex sets. Conjugate of a convex function, support function of a convex set polar of a convex set.. Infimal -convolution and functional conjugacy

##### **IV. Subdifferential calculus for convex functions**

Directional derivative and subgradient. Subdifferential and monotonicity. Derivability of convex functions.

##### **V. Duality in convex optimization**

Optimality for a convex program, Kuhn-Tucker conditions. Programme convexe Ordinary convex programs and Lagrangian multipliers . Lagrange duality and Fenchel duality.

## **VI. Some basic algorithms in convex optimization.**

Reduced gradient algorithm, conjugate gradient algorithm and Lemke method for convex quadratic programs. Subgradient projection method, Kelley's cutting plane method for nondifferentiable convex programs. Maximal monotone operators and proximal algorithms: Applications to decomposition in convex programming and Augmented Lagrangian methods.. Interior point methods in convex programming. Newton methods

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## **B. MODELIZATION, OPTIMIZATION NON CONVEXE & GLOBAL OPTIMIZATION**

**I. Classification of nonconvex programs :** Convex maximization , DC programs (minimization of a DC function on a convex set), minimization of a convex function on an intersection of a convex set and a complement of a convex set.

## **II. DC Programming and DCA**

Structure of the space of DC functions. DC duality, DC programs primal and dual. Relations between primal and dual solutions. Local optimality and global optimality in DC programming. Regularization in DC programming. Exact penalty and reformulation in DC Programming.

## **III. DC Algorithms (DCA)**

Description and interpretation of DCA. Choice of DC decompositions in DC programming.

Standard algorithms in convex (resp. nonconvex) programming as special cases of DCA

Modeling and solution of real-world nonconvex programs by DCA

## **IV. Algorithms for solving nonconvex programs**

Branch and Bound techniques (BB) for globally solving DC programs

Combination of DCA and BB : Applications to globally solving mixed 0-1 nonconvex quadratic programs and real-world nonconvex programs

Trust region methods for solving nonconvex programs.

**V. Applications:** modelling of different problems in various areas: (data mining, bioinformatic, signal and image processing, Computer security, Engineering structures, finance,

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