Exercises: Number Theory

1. Show that $3^n \equiv -1(\text{mod}10) \Leftrightarrow 3^{n+4} \equiv -1(\text{mod } 10)$.
2. Find the remainder in the division algorithm with divisor 9 and dividend $1532^5 - 1$.
3. Show that
   $2^{3^n} \equiv -1(mod\ 3^{n+1})$.
4. Show that
   $102|(220^{119^{69}} + 119^{69^{220}} + 69^{220^{119}})$.

5. How many integer between 100 and 1000 are divisible by 7? by 49 ?
6. Convert 1864 from decimal to base 2.
7. Convert $(1999)_{10}$ from decimal to base 7 notation.

   Convert $(6105)_7$ from 7 to decimal notation.

8. Convert $(101001000)_2$ from binary to decimal notation and $(1984)_{10}$ from decimal to binary notation.
9. Find the greatest common divisor of each of the following pairs of integers

   a) 15 , 35 ; b) 0 , 111 ; c) -12 , 18 ;
   d) 99 , 100 ; e) 11, 121 ; f) 100, 102 ;
   g) 5 , 15 ; h) -90 , 100 ; i) 100 ,121 ;
   k) 1001 , 289 ; l) 20785 , 44350 ;
   m) 34709 , 100313.

10. Let a be a positive integer .What is the greatest common divisor of a and a +2 .
11. Show that if a and b are positive integers , then [a , b] = ab / (a , b),where [a, b] and

    (a, b) are the least common multiple and greatest common divisor of a and b ,respectively.

12. Find all solutions of each of the following linear congruences
    a) $2x \equiv 5 \pmod 7$; b) $3x \equiv 6 \pmod 9$; c) $19x \equiv 30 \pmod{40}$;

    d) $9x \equiv 5 \pmod{25}$; e) $103x \equiv 444 \pmod{999}$;

    f) $980x \equiv 1500 \pmod{1600}$.

13. Find all solutions of each of the following linear congruences

    a) $3x \equiv 2 \pmod 7$; b) $15x \equiv 9 \pmod{25}$; c) $6x \equiv 3 \pmod 9$;

    d) $128x \equiv 833 \pmod{1001}$; e) $17x \equiv 14 \pmod{21}$; f) $987x \equiv 610 \pmod{1597}$.

14. Find all solutions to the congruence

6789783x ≡ 2474010 (mod 28937591).

15. Find the solution of each of the following systems of linear congruences

$a)\begin{cases}3x+4y \equiv 5(mod13)\\ 2x+5y \equiv 7(mod13)\end{cases}$ $b)\begin{cases}x+2y \equiv 1\ (mod5)\\ 2x+y \equiv 1\ (mod5)\end{cases}$ $c)\begin{cases}x+3y \equiv 1(mod5)\\ 3x+4y \equiv 2(mod5)\end{cases}$

16. Find all the quadric residues of the following integers

a) 3 ; b) 5 ; c) 13 ; d) 19 ; e) 7.

17. Find the value of the Legendre symbols $\left(\frac{j}{5}\right)$ for j = 1, 2, 3, 4 .

18. Find the value of the Legendre symbols $\left(\frac{j}{7}\right)$ for j = 1, 2, 3, 4 , 5 , 6.

19. Evaluate the Legendre symbol $\left(\frac{7}{11}\right)$ using Euler's criterion.

20. Find the number of incongruent solutions of each the following congruences :

a) $x^2 \equiv 741$ (mod 1597).

b) $x^2 \equiv 219$ (mod 383).