TN326 GAME THEORY

- www.math.hcmuns.edu.vn/~nvthuy/TN326.html
- Tài liệu tham khảo
  - [1] Thomas S. Ferguson, *Game Theory*, UCLA, 2005.(PDF)
  - [2] Guillermo Owen, *Game Theory*, Academic Press, 1995.
- Điểm môn học
  - Bài tập: 20%
  - Kiểm tra giữa học kỳ: 30%
  - Thi cuối kỳ: 50%

1. Introduction to Games

- What is game theory ?
- Examples
- Representation of game
- Types of games
- Application of game theory

1.1 What is game theory ?

- Ten of you go to a restaurant
- If each of you pays for your own meal…
  - This is a decision problem
- If you all agree to split the bill...
  - Now, this is a game

1.1 What is game theory ?

- studies decisions made in an environment in which players interact
- studies choice of optimal behavior when costs and benefits of each option depend upon the choices of other individuals
- J. von Neumann and O.Morgenstern (1944) *The Theory of Games and Economic Behavior*, Princeton University Press.
- Nobel Prize:
  - 1994: John Nash,
  - 2005: Thomas Schelling, Robert Aumann

1.1 What is game theory ?

- Players
  - Everyone who has an effect on your earnings
- Strategies
  - Actions available to each player
  - Define a plan of action for every contingency
- Payoffs
  - Numbers associated with each outcome
  - Reflect the interests of the players

1.1 What is game theory ?

- Timing of moves
  - Are moves simultaneous or sequential?
- Nature of conflict and interaction
  - Are players' interests in conflict?
  - Will players interact once or repeatedly?
- Informational conditions
  - Are some players better informed?
- Enforceability of agreements
  - Can contracts be enforced?

1.2 The Prisoner's Dilemma

- If neither confesses then both will be convicted of a minor offense and sentenced to one month in jail.
- If both confess then both will be sentenced to jail for 3 months.
- If one confesses but the other does not, then the confessor will be released but the other will be sentenced to jail for 4 months.

Prisoner 1 \ Prisoner 2	cooperate	defect
cooperate	3 , 3	0 , 4
defect	4 , 0	1 , 1

1.2 Matching pennies

- Each of the two players has a penny.
- Two players must simultaneously choose whether to show the Head or the Tail.
- Both players know the following rules:
  - If two pennies match (both heads or both tails) then player 2 wins player 1's penny.
  - Otherwise, player 1 wins player 2's penny.

		Player 2: Head	Player 2: Tail
Player 1	Head	-1 , 1	1 , -1
	Tail	1 , -1	-1 , 1

1.2 Games of Chicken

		Firm 2: Stay	Firm 2: Swerve
Firm 1	Stay	-50 , -50	100 , 0
	Swerve	0 , 100	50 , 50

1.2 Tourists & Natives

- Only two bars (bar 1, bar 2) in a city
- Can charge price of $2, $4, or $5
- 6000 tourists pick a bar randomly
- 4000 natives select the lowest price bar

- Example 1: Both charge $2
  - each gets 5,000 customers and $10,000
- Example 2: Bar 1 charges $4, Bar 2 charges $5
  - Bar 1 gets 3000+4000=7,000 customers and $28,000
  - Bar 2 gets 3000 customers and $15,000

1.2 One More Example

- Each of n players selects a number between 0 and 100 simultaneously. Let x_i denote the number selected by player i.
- Let y denote the average of these numbers
- Player i's payoff = $\mathbf{x_i - 3y/5}$

1.3 Representation of game

- Normal form
- Extensive form

Normal-form or strategic-form

- The *normal-form* (*or strategic-form*) *representation* of a game G specifies:
  - A finite set of players $\{1, 2, ..., n\}$,
  - players' strategy spaces $S_1 \ S_2 \ \ldots \ S_n$ and
  - their payoff functions $u_1 \ u_2 \ \ldots \ u_n$ where $u_i : S_1 \times S_2 \times \ldots \times S_n \rightarrow R$.

Normal-form: 2-player game

- Bi-matrix representation
  - 2 players: Player 1 and Player 2
  - Each player has a finite number of strategies
- Example:

$S_1=\{s_{11}, s_{12}, s_{13}\}$ $S_2=\{s_{21}, s_{22}\}$

Player 2

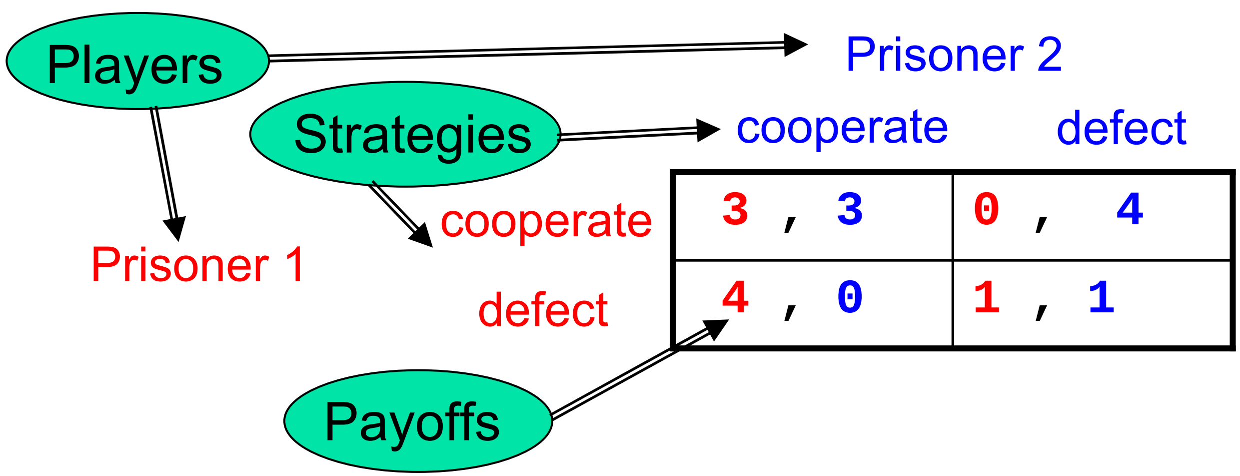
Player 1		s_{21}	s_{22}
	s_{11}	$u_1(s_{11},s_{21}), u_2(s_{11},s_{21})$	$u_1(s_{11},s_{22}), u_2(s_{11},s_{22})$
	s_{12}	$u_1(s_{12},s_{21}), u_2(s_{12},s_{21})$	$u_1(s_{12},s_{22}), u_2(s_{12},s_{22})$
	s_{13}	$u_1(s_{13},s_{21}), u_2(s_{13},s_{21})$	$u_1(s_{13},s_{22}), u_2(s_{13},s_{22})$

Prisoners' Dilemma in normal-form

- Set of players: {Prisoner 1, Prisoner 2}
- Sets of strategies: $S_1 = S_2 =$ **{Cooperate, Defect}**
- Payoff functions:

u_1(C, C)=3, u_1(C, D)=0, u_1(D, C)=4, u_1(D, D)=1;

u_2(C, C)=3, u_2(C, D)=4, u_2(D, C)=0, u_2(D, D)=1



	cooperate	defect
cooperate	3 , 3	0 , 4
defect	4 , 0	1 , 1

Example: Matching pennies

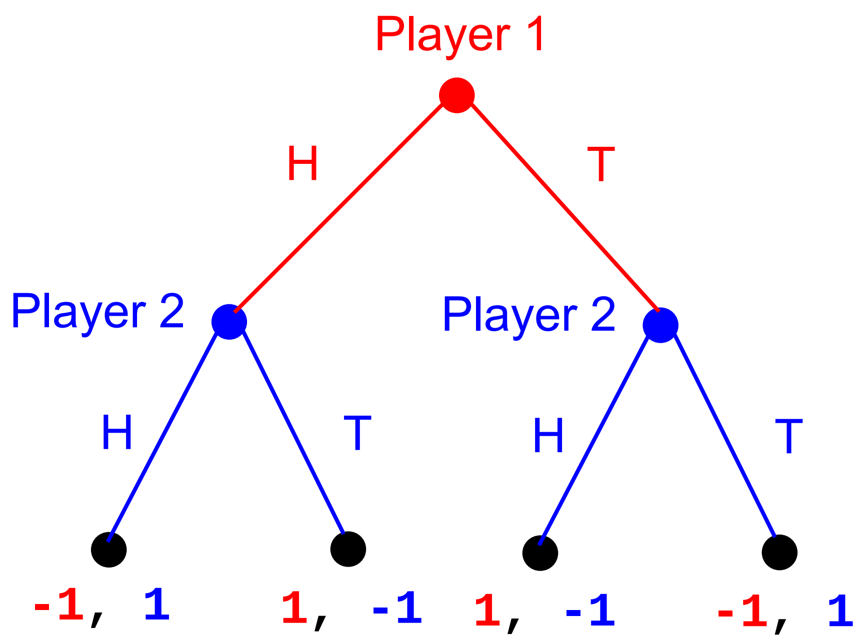
		Player 2 Head	Player 2 Tail
Player 1	Head	-1 , 1	1 , -1
	Tail	1 , -1	-1 , 1

- Normal (or strategic) form representation:
  - Set of players: {Player 1, Player 2}
  - Sets of strategies: $S_1 = S_2 = \{ \underline{H}ead, \underline{T}ail \}$
  - Payoff functions:

  $u_1(H, H)=-1$, $u_1(H, T)=1$, $u_1(T, H)=1$, $u_1(H, T)=-1$;
  $u_2(H, H)=1$, $u_2(H, T)=-1$, $u_2(T, H)=-1$, $u_2(T, T)=1$

Extensive form

- Each of the two players has a penny.
- Player 1 first chooses whether to show the Head or the Tail.
- After observing player 1's choice, player 2 chooses to show Head or Tail
- Both players know the following rules:
  - If two pennies match (both heads or both tails) then player 2 wins player 1's penny.
  - Otherwise, player 1 wins player 2's penny.



1.4 Types of games

- Cooperative or noncooperative
- Symmetric and asymmetric
- Zero sum and non-zero sum
- Simultaneous and sequential
- Perfect information and imperfect information
- Infinitely long games

1.5 Application of game theory

- Political science
- Economics and business
- Biology
- Computer science and logic
- Philosophy