Sujet de stage pour le Master de Mathématiques
Orléans

Titre : Solutions analytiques pour les équations de Saint-Venant

Responsables : Stéphane Cordier, Olivier Delestre & François James

Coordonnées : Université d’Orléans & CNRS
Fédération Denis Poisson (FR 2964), Laboratoire MAPMO (UMR 6628)
Bâtiment de Mathématiques, BP 6759, 45067 Orléans cedex 2, France
Contact email : cordier@math.cnrs.fr

Abstract :
The shallow water (or Saint-Venant) equations are hyperbolic partial differential
equations which are used to simulate water flows when the depth of water is not
too large. In one space dimension, they read
\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} &= 0 \\
\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2 + gh^2/2)}{\partial x} + h\frac{\partial Z}{\partial x} &= -ghS_f,
\end{align*}
\]
where \(h(t, x)\) is the water height, \(u(t, x)\) the flow velocity, \(z(x)\) the topography,
\(S_f\) a friction term.

Searching for analytical solutions is important, in order to have a better
insight of the problem, and also to obtain test cases for general numerical meth-
ods (e.g. finite volumes [B]). Several configurations are known, but they do not
take into account friction terms. The aim of this project is, starting from these
classical tests ([GHS], [H], [VCEL]), to proceed towards new analytical solutions
to this system, including friction terms.

Some abilities in using formal calculus software (such as Maple) are expected
since the complete formulation of these analytical solutions contains nonlinear
algebraic equations, which have to be solved, for instance by formal calculus.

This training course is part of a research program, called METHODE, funded
by the National Research Agency in France (ANR).
See http://www.univ-orleans.fr/mapmo/methode for more details on this project.

References :
[GHS] T. Gallouët, J.-M. Hérard, N. Seguin, Some Godunov schemes to compute
[H] J.M. Hervouet, Hydrodynamique des écoulements à surface libre, modélisation
numérique avec la méthode des éléments finis, Presses des Ponts et
conservation laws, and well-balanced schemes for sources, Birkhauser (2004).